AP Calculus AB

Review Week 6

Particle Motion, Differential Equations and the rest

Advanced Placement AAP Review will be held in **room 315** and **312** on Tuesdays and Thursdays.

The week of April 27th we will be reviewing **Differential Equations.**

The session will begin in room 315 with a brief review of the weekly topic.

Instruction will be from 3:00 pm to 3:15 pm

Once we have reviewed the topic you may begin practicing the questions in your review packet.

Answers will be posted in room 315 and 312 all week and will be posted on line after 3:00 pm on Friday the week of review.

If you have difficulty with a question look at the detailed answer postings BEFORE you ask your teacher for help.

Get a hint....**<u>DON'T COPY THE ANSWER!!! THAT IS NOT HELPFUL!!</u>**

When you have completed a question...**<u>REFLECT!!!!</u>** Ask yourself what skill you used to solve that problem and write that down!!

Once we have completed the weekly review, keep it to study from as we get closer to the exam.

Session Notes

Suppose an object is moving along a straight line, such as the x-axis, so that its position x, as a function of time t, on that line is given by $y = x(t)$.

Average velocity of the object over the time interval t to $t + \Delta t$ is given by
$\dfrac{x(t+\Delta t) - x(t)}{\Delta t}$, or $\dfrac{\text{change in position}}{\text{change in time}}$.

Instantaneous velocity of the object is the derivative of the position function $x(t)$ with respect to time. $v(t) = x'(t)$

Speed is the absolute value of the velocity. $Speed = |v(t)| = \left|\dfrac{dx}{dt}\right|$.

Acceleration is the derivative of velocity with respect to time. $a(t) = v'(t) = x''(t)$

$\int v(t)\,dt = x(t) + c$,

$\int a(t)\,dt = v(t) + c$

Total distance traveled from time $t = t_1$ *to* $t = t_2$ is given by

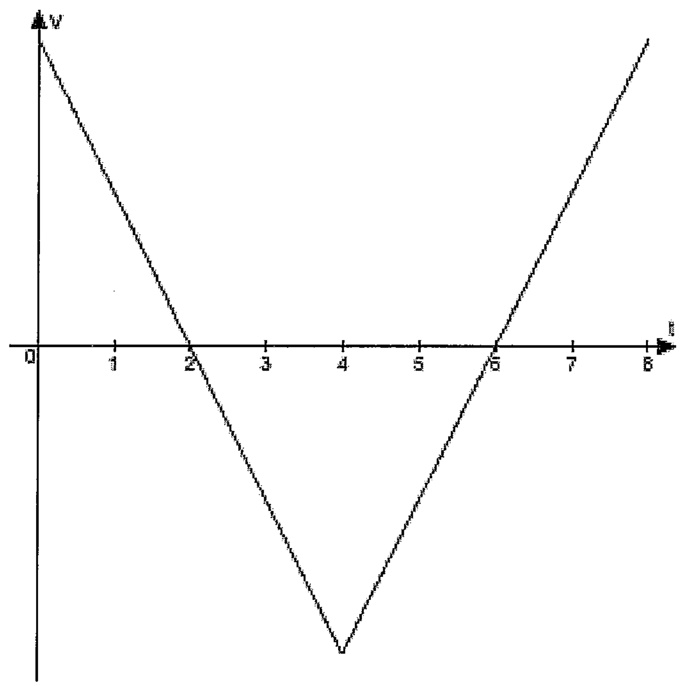
$TDT = \int_{t_1}^{t_2} |v(t)|\,dt$.

Speeding Up or Slowing Down

If the velocity and acceleration have the same sign (both positive or both negative), then speed is increasing. If an object's velocity is -40 miles per hour and the object accelerates -10 miles per hour per hour, the object is speeding up.

If the velocity and acceleration are opposite in sign (one is positive and the other is negative), then speed is decreasing. If an object's velocity is -40 miles per hour and the object accelerates 10 miles per hour per hour, the object is slowing down.

Sign Convention: When the object is moving in the right direction or moving upward then the velocity is positive (Graph of velocity vs. time is above the t axis). When the object is moving in the left direction or moving downward then the velocity is negative. A graph of velocity vs. time is shown below.



$0<t<2$	$v(+), a(-)$	Object is slowing down
$2<t<4$	$v(-), a(-)$	Object is speeding up
$4<t<6$	$v(-), a(+)$	Object is slowing down
$6<t<8$	$v(+), a(+)$	Object is speeding up

What you need to know about motion along the x-axis:

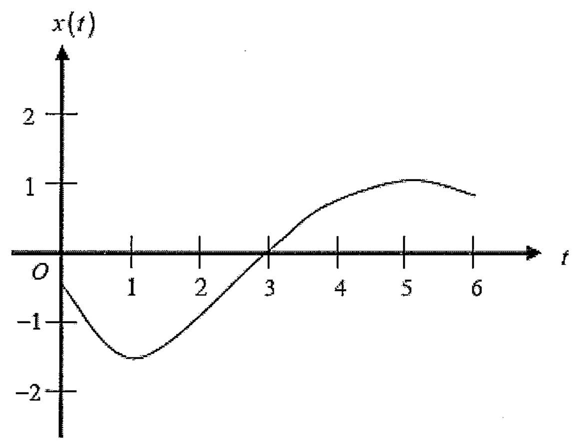
When you see…	Think…
Initially	$t=0$
At rest	$v(t)=0$
At the origin	$x(t)=0$
Velocity is positive	Particle is moving right (or up)
Velocity is negative	Particle is moving left (or down)
Average velocity (Given $x(t)$)	Change in position divided by change in time
Average velocity (Given $v(t)$)	$\frac{1}{b-a}\int_a^b v(t)\,dt$ (The average value of the velocity function.)
Instantaneous velocity	Velocity at an exact moment
Positive acceleration	Velocity is increasing
Negative acceleration	Velocity is decreasing
Speed	$\lvert v(t)\rvert$
Total Distance Traveled	$\int_a^b \lvert v(t)\rvert\,dt$

2008 Non Calculator

7. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at time $t = 0$, what is the position of the particle at $t = 1$?

(A) 4 (B) 6 (C) 9 (D) 11 (E) 12

[redacted]



21. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown above for $0 < t < 6$. The graph has horizontal tangents at $t = 1$ and $t = 5$ and a point of inflection at $t = 2$. For what values of t is the velocity of the particle increasing?

(A) $0 < t < 2$

(B) $1 < t < 5$

(C) $2 < t < 6$

(D) $3 < t < 5$ only

(E) $1 < t < 2$ and $5 < t < 6$

[redacted]

23. Which of the following is the solution to the differential equation $\frac{dy}{dx}=\frac{x^2}{y}$ with the initial condition $y(3)=-2$?

(A) $y=2e^{-9+x^3/3}$

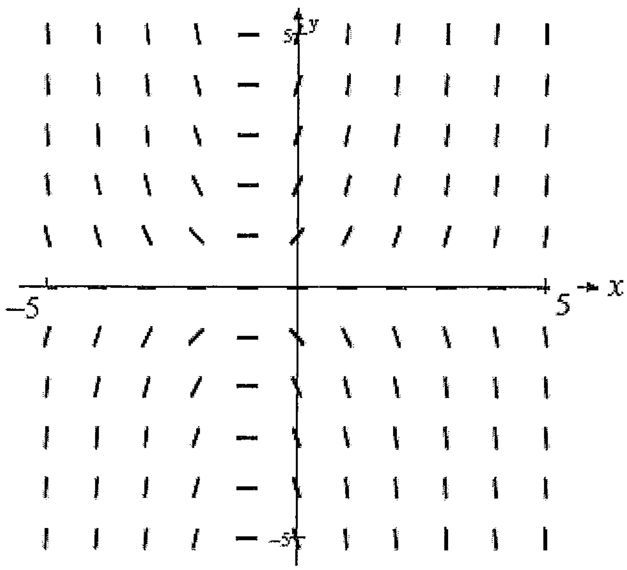
(B) $y=-2e^{-9+x^3/3}$

(C) $y=\sqrt{\frac{2x^3}{3}}$

(D) $y=\sqrt{\frac{2x^3}{3}-14}$

(E) $y=-\sqrt{\frac{2x^3}{3}-14}$

[redacted]



27. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = xy$

(B) $\frac{dy}{dx} = xy - y$

(C) $\frac{dy}{dx} = xy + y$

(D) $\frac{dy}{dx} = xy + x$

(E) $\frac{dy}{dx} = (x+1)^3$

27. Slope field:

[redacted]

[redacted] $\therefore$ [redacted]

2008 Calculator

82. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

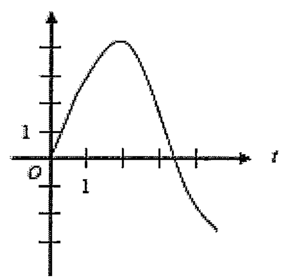
(A) −0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

[redacted]

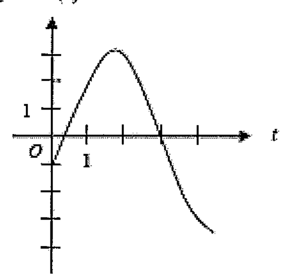
t	0	1	2	3	4
$v(t)$	−1	2	3	0	−4

86. The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \leq t \leq 4$?

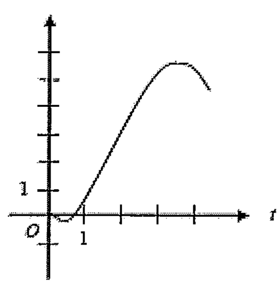
(A) $x(t)$



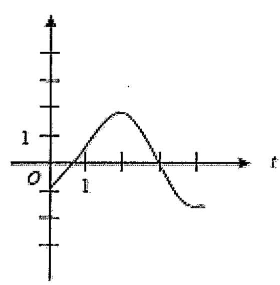
(B) $x(t)$



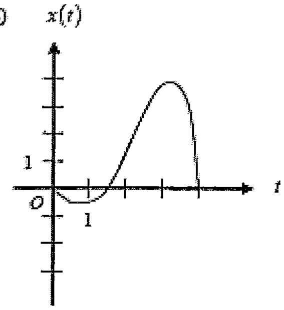
(C) $x(t)$



(D) $x(t)$



(E) $x(t)$



[redacted]

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

[redacted]

2003 Non Calculator

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only

(B) $t = 3$ only

(C) $t = \frac{7}{2}$ only

(D) $t = 3$ and $t = \frac{7}{2}$

(E) $t = 3$ and $t = 4$

[redacted]

2003 Calculator

76. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = 3 + 4.1\cos(0.9t)$. What is the acceleration of the particle at time $t = 4$?

(A) −2.016 (B) −0.677 (C) 1.633 (D) 1.814 (E) 2.978

[redacted]

83. The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time $t = 0$ to time $t = 3$?

(A) 20.086 ft/sec

(B) 26.447 ft/sec

(C) 32.809 ft/sec

(D) 40.671 ft/sec

(E) 79.342 ft/sec

[redacted]

91. A particle moves along the x-axis so that at any time $t > 0$, its acceleration is given by $a(t) = \ln(1 + 2^t)$. If the velocity of the particle is 2 at time $t = 1$, then the velocity of the particle at time $t = 2$ is .

(A) 0.462 (B) 1.609 (C) 2.555 (D) 2.886 (E) 3.346

[redacted]

2013 Open Ended #1

1. On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

2. A particle moves along a straight line. For $0 \leq t \leq 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \leq t \leq 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \leq t \leq 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

2012 Open Ended

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

   (a) For $0 \le t \le 12$, when is the particle moving to the left?

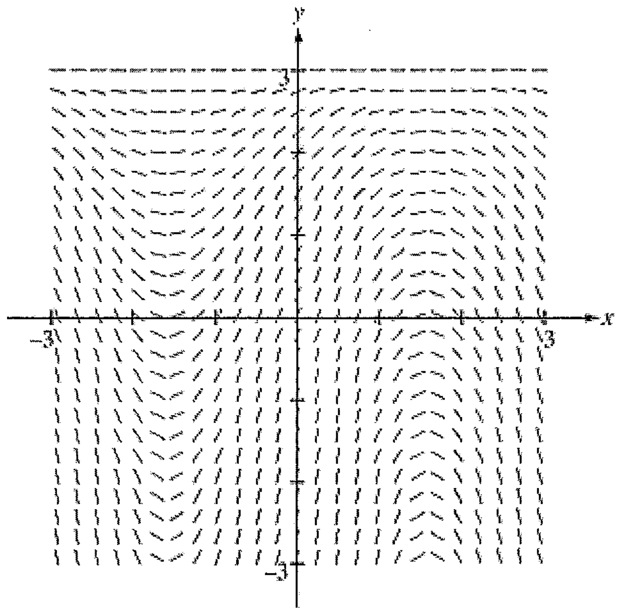
   (b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

   (c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

   (d) Find the position of the particle at time $t = 4$.

6. Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos x$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

   (a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



   (b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

   (c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 1$.